An Analysis of the Blue Noise Mask Based on a Human Visual Model

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Department of Electrical Engineering
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ABSTRACT

The Blue Noise Mask (BNM) is a stochastic screen that produces visually pleasing blue noise. In its construction, a [illegible]

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$$E'^2 = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[f(i,j) + h\left((i-i_0)_{\mathrm{mod}\,N},(j-j_0)_{\mathrm{mod}\,N}\right) - g'\right]^2$$

$$= \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left[f(i,j) - g'\right]^2 + \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} h^2\left((i-i_0)_{\mathrm{mod}\,N},(j-j_0)_{\mathrm{mod}\,N}\right)$$

$$+ 2\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} f(i,j)h\left((i-i_0)_{\mathrm{mod}\,N},(j-j_0)_{\mathrm{mod}\,N}\right)$$

$$- 2\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} g'h\left((i-i_0)_{\mathrm{mod}\,N},(j-j_0)_{\mathrm{mod}\,N}\right) \tag{8}$$

There are four terms on the right side of the above equation. Since g' is a constant and f(i,j) is known, the first term in equation (8) is fixed. The second term is a summation of the shifted filter squared over the support of the BNM. Due to the "wrap-around" property, this term is constant. For the same argument, the fourth term is also a constant. Let

$$G(i_0,j_0) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} f(i,j)h\left((i-i_0)_{\mathrm{mod}\,N},(j-j_0)_{\mathrm{mod}\,N}\right) \tag{9}$$

where $G(i_0,j_0)$ is a function of i_0 and j_0. It follows that minimizing E'^2 is reduced to minimizing $G(i_0,j_0)$.

Substituting (3) into (9) we have:

$$G(i_0,j_0) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} h\left((i-m)_{\mathrm{mod}\,N},(j-n)_{\mathrm{mod}\,N}\right)b(m,n)h\left((i-i_0)_{\mathrm{mod}\,N},(j-j_0)_{\mathrm{mod}\,N}\right) \tag{10}$$

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$$R\left(m,n,i_0,j_0\right)=\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} h(k,l)h\Big(\big(k+(m-i_0)\big)_{\mathrm{mod}\,N},\big(l+(n-j_0)\big)_{\mathrm{mod}\,N}\Big) \tag{15}$$

which is the circular autocorrelation (for simplicity, we will use "autocorrelation" for "circular autocorrelation" from now on) of h at $\left(m-i_0, n-j_0\right)$ and can be denoted as $R_h\Big(\left(m-i_0\right)_{\mathrm{mod}\,N},\left(n-j_0\right)_{\mathrm{mod}\,N}\Big)$. Substituting into (11) and making use of the symmetry of the autocorrelation function, we obtain:

$$G\left(i_0,j_0\right)=\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} b(m,n)R_h\Big(\left(i_0-m\right)_{\mathrm{mod}\,N},\left(j_0-n\right)_{\mathrm{mod}\,N}\Big) \tag{16}$$

In the frequency domain, R_h corresponds to $|H|^2$. It can be easily seen that $G\left(i_0,j_0\right)$ is the circular convolution of the binary pattern for level g with a new filter that is the autocorrelation function of h.

Examination of equation (11) tells us that to minimize $G\left(i_0,j_0\right)$, or the perceived error for level g' measured by the

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Let's examine how we can find the P black points in f(i,j) which, when converted to white, will minimize E'^2, or $G(i_0, j_0)$. To minimize the first term of $G(i_0, j_0)$, we use the autocorrelation function R_h of h as a filter and apply it